Int. J. Solids Structures Vol. 26, No. 3, pp. 299–312, 1990
Printed in Great Britain.

0020–7683/90 $3.00 + .00

STATICAL AND DYNAMICAL RESPONSE OF CABLE NETS

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(*Received* 9 *July* 1987 ; *in final form* 22 *June* 1989)

Abstract—The statical and the dynamical behaviour of cable nets is studied. Fully constrained and underconstrained cable nets are considered. The conditions for initial stability and the prestressing of the nets are formulated. A method for the analysis of the nodal displacements and the change in the internal forces in the cables due to static loads is proposed. The cases where linear algebra can be used are discussed. The method is extended to consider the dynamical behaviour of the net. Its free dynamical response and forced vibration are investigated.

INTRODUCTION

An extensive amount of work has been done in studying the nodal displacements and the change of the internal forces of cable nets due to statical and dynamical loads. The difficulty of the analysis lies in the fact that the system is geometrically non-linear. Intricate non-linear methods in which the problem is analysed step by step are proposed for this purpose ; see Buchholdt *et al.* (1968), Irvine (1981) and Otto (1966). Some commercial computer programs based on these methods are available on the market. There have been a few attempts, by one of the authors and others, to develop an approximate linear method of analysis ; see Vilnay (1981), Vilnay and Soh (1982), Calladine (1982), Pellegrino and Calladine (1984), Vilnay (1985), Pellegrino and Calladine (1986) and Vilnay (1987).

In this work the linear method is developed to consider the non-linear effect. The final nodal displacements are considered and the governing equations are formulated. The cases and loads in which the displacements of the net are small and linear algebra is accurate are formulated. The loads in which some of the linear equations should be replaced by higher order algebraic equations are discussed. The fact that it is easy to detect where the net displacements are small and linear algebra is accurate is of major advantage. It makes the analysis very simple and even where large number of nodes are considered it imposes no major difficulties. Also in the cases where the nodal displacements are large and some of the equations are of high order the proposed formulation gives the possibility of solving the problem quicker and more efficiently than the step by step methods in which the load is increased incrementally at each step.

The method is developed to consider the dynamical response of the net. It is shown that in underconstrained cable nets the free vibration is associated with two types of frequencies, high and low. A simple method is proposed for the analysis of these frequencies. It is shown that the net free dynamical displacements are governed by the low frequencies whereas the change in the internal forces is governed by the high frequencies. The effect of dynamical imposed load is studied and the cases where small displacements are expected are discussed and formulated.

PRESTRESSING AND INITIAL STABILITY

From an engineering point of view every cable net used in structures should satisfy the following initial stability requirement.

All cables should be straight under all expected loads.

In order to satisfy this requirement there should be a certain amount of tension in all cables always. Thus a satisfactory cable net should be :

(i) prestressable,
(ii) prestressing induces tension in all cables.

The level of prestressing is that needed to keep a required minimum amount of tension in all cables under all expected loads.

The net is prestressable and condition (i) is satisfied where all nodes, considering the forces induced into the net numbers by prestressing, are in equilibrium. The nodal equilibrium of the net takes the form :

$$\tilde{A}\mathbf{P} = 0 \tag{1}$$

in which $\tilde{A}$ is the equilibrium matrix, a function of the net geometrical configuration and $\mathbf{P}$ is the vector of the forces induced by prestressing. Equation (1) is satisfied where :

$$r < m \tag{2}$$

in which r is the rank of matrix $\tilde{A}$ and m is the number of the unknown forces induced by prestressing into the net and/or the number of columns of matrix $\tilde{A}$.

A cable net is fully constrained where :

$$r = n \quad \text{and} \quad m > n \tag{3}$$

in which n is the number of equilibrium equations and/or the number of rows of matrix $\tilde{A}$.

In this case matrix $\tilde{A}$ and vector $\mathbf{P}$ can be partitioned into :

$$\tilde{A} = [\tilde{A}^0 \quad \tilde{A}^g] \; ; \quad \mathbf{P} = \begin{bmatrix} \mathbf{P}^t \\ \mathbf{P}^b \end{bmatrix} \tag{4}$$

in which matrix $\tilde{A}^0$ is a square matrix of $r \times r$ established from matrix $\tilde{A}$ with $\det \tilde{A}^0 \neq 0$. $\tilde{A}^g$ is an $r \times (m-r)$ matrix composed of the rest of the elements of matrix $\tilde{A}$ properly arranged. $\mathbf{P}^t$ is composed of the r elements of vector $\mathbf{P}$ associated with $\tilde{A}^0$ and $\mathbf{P}^b$ is composed of the rest of the elements of vector $\mathbf{P}$ associated with $\tilde{A}^g$.

A cable net is underconstrained where :

$$r < n. \tag{5}$$

In this case matrix $\tilde{A}$ and vector $\mathbf{P}$ can be partitioned into :

$$\tilde{A} = \begin{bmatrix} \tilde{A}^0 & \tilde{A}^g \\ \tilde{A}^d & \tilde{A}^e \end{bmatrix} ; \quad \mathbf{P} = \begin{bmatrix} \mathbf{P}^t \\ \mathbf{P}^b \end{bmatrix} \tag{6}$$

in which matrices $\tilde{A}^d$ and $\tilde{A}^e$ are appropriate $(n-r) \times r$ and $(n-r) \times (m-r)$ matrices, respectively. Inequality (2) implies a linear relationship between the elements of matrix $\tilde{A}$:

$$\begin{aligned} \tilde{A}^d &= \tilde{V}\tilde{A}^0 \; ; \quad (\tilde{A}^0)^{\mathrm{T}}\tilde{V}^{\mathrm{T}} = (\tilde{A}^d)^{\mathrm{T}} \\ \tilde{A}^e &= \tilde{V}\tilde{A}^g \; ; \quad (\tilde{A}^g)^{\mathrm{T}}\tilde{V}^{\mathrm{T}} = (\tilde{A}^e)^{\mathrm{T}} \end{aligned} \tag{7}$$

in which $\tilde{V}$ is the transformation matrix of $(n-r) \times r$ and $(\;)^{\mathrm{T}}$ indicates the matrix transposed. By using inequalities (2) and (7), matrix $\tilde{V}$ takes the form :

$$\tilde{V} = \tilde{A}^d(\tilde{A}^0)^{-1}. \tag{8}$$

The forces induced by prestressing into the elements can be determined by using eqn

(1). The number of independent degrees of freedom of the prestressing is equal to the order of vector $\mathbf{P}^b$. In the case where $\mathbf{P}^b$ takes the form :

$$\mathbf{P}_1^b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \end{bmatrix} \tag{9}$$

the forces induced into the net takes the form :

$$\mathbf{P}_1 = \begin{bmatrix} \mathbf{P}_1^t \\ \mathbf{P}_1^b \end{bmatrix} \tag{10}$$

in which

$$\mathbf{P}_1^t = -(\tilde{A}^0)^{-1}\tilde{A}^g\mathbf{P}_1^b.$$

By following the same method $\mathbf{P}_2^b$ can be established :

$$\mathbf{P}_2^b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \end{bmatrix} \tag{11}$$

and by using eqn (1), $\mathbf{P}_2^t$ and $\mathbf{P}_2$ can be determined. By grouping vectors $\mathbf{P}_i$ the prestressing matrix $\tilde{P}$ of the order $m \times (m-r)$ can be established. Condition (ii) is satisfied where it is possible to find a vector $\mathbf{D}$ in which all elements are positive of the order $(m-r)$ so that :

$$\tilde{P}\mathbf{D} > 0. \tag{12}$$

The elements of vector $\tilde{P}\mathbf{D}$ indicate the magnitude of the forces induced in the elements of the cable net due to prestressing.

The net shown in Fig. 1 is composed of five cables intersecting at six nodes. Matrix $\tilde{A}$

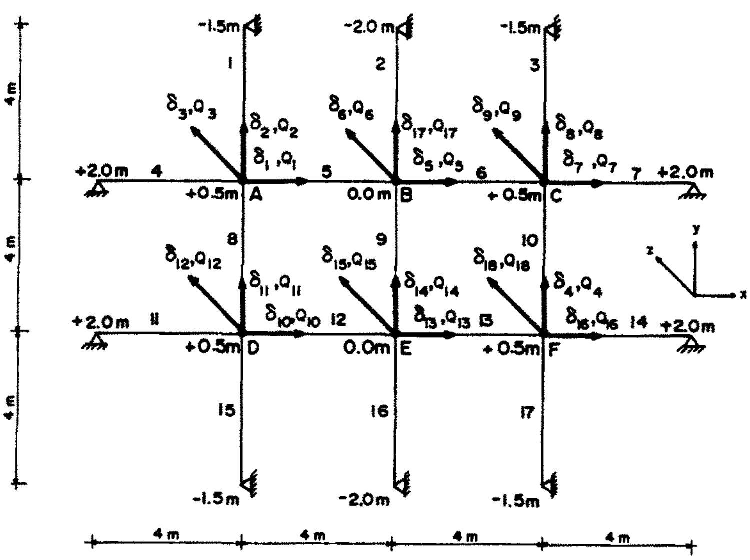

Fig. 1. A cable net.

of this net takes the form:

$$
\tilde{A} = \begin{array}{c}
\tilde{A}^0 \qquad\qquad\qquad\qquad \tilde{A}^g \\
\left[\begin{array}{cccccccccccccccc|c}
0 & 0 & 0 & -0.937 & 0.993 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.894 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.417 & 0 & 0 & 0.351 & -0.124 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.894 \\
0 & 0 & 0 & 0 & -1.0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.447 & 0 & 0 & 0.124 & 0.124 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0 & -0.993 & 0.937 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.894 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.447 & 0 & 0 & -0.124 & 0.351 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.937 & 0.993 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.894 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.351 & -0.124 & 0 & 0 & -0.447 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0 & 1.0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.894 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.124 & 0.124 & 0 & 0 & -0.447 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.993 & 0.937 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.124 & 0.351 & 0 & 0 & -0.447
\end{array}\right] \\
\tilde{A}^d \qquad\qquad\qquad\qquad A^e
\end{array} . \tag{13}
$$

The rank of matrix $\tilde{A}$ is 16, thus inequality (2) is satisfied and this cable net is underconstrained. Matrix $\tilde{A}$ is partitioned into $\tilde{A}^0$, $\tilde{A}^g$, $\tilde{A}^d$ and $\tilde{A}^e$, as shown in eqn (13). By using eqn (7), matrix $\tilde{V}$ takes the form:

$$\tilde{V} = \begin{bmatrix} 0.749 & 1.0 & 2.0 & 0.0 & 0.248 & -2.0 & 0.0 & 0.0 & 0.0 & -0.749 & 1.0 & -2.0 & -0.248 & -1.0 & 2.0 & 0.0 \\ -0.375 & -0.5 & -1.0 & 0.5 & -0.248 & 0.0 & -0.375 & 0.5 & 1.0 & 0.375 & -0.5 & 1.0 & 0.248 & 0.0 & 0.0 & 0.375 \end{bmatrix}. \quad (14)$$

Equation (6) implies that there is one element in vector $\mathbf{P}^b$. By using eqn (9), the prestressing matrix takes the form of a vector:

$$\tilde{P}^{\mathrm{T}} = [1.00 \quad 1.00 \quad 1.00 \quad 1.93 \quad 1.82 \quad 1.82 \quad 1.93 \quad 0.91 \quad 0.91 \quad 0.91 \quad 1.93 \quad 1.82 \quad 1.82 \quad 1.93 \quad 1.00 \quad 1.00 \quad 1.00]. \quad (15)$$

It can be seen that in the case where $\mathbf{D} = a$, where $a > 0$, eqn (12) is satisfied. Where $\mathbf{D} = 20.0$ kN, the prestressing forces in the cables are:

$$\mathbf{P} = 20\tilde{P}\,\mathrm{kN}. \quad (16)$$

STATIC LOADS

In the case where the cable net is loaded by static load $\mathbf{Q}$ imposed at the net nodes the nodal equilibrium given by eqn (1) takes the form:

$$\tilde{A}(\mathbf{P}+\mathbf{F})+\mathbf{Q} = 0 \quad (17)$$

in which $\mathbf{F}$ indicates the change in the internal forces in the prestressed cables due to the static load. By using eqn (1), eqn (17) takes the form:

$$\tilde{A}\mathbf{F}+\mathbf{Q} = 0. \quad (18)$$

In the case of a fully constrained cable net, eqn (18) is a consistent set of linear equations. It can be solved by considering the relationship between the nodal displacements $\boldsymbol{\delta}$ and the change in the internal forces (Livesley, 1969):

$$\mathbf{F} = \tilde{K}\tilde{A}^{\mathrm{T}}\boldsymbol{\delta} \quad (19)$$

where $\tilde{K}$ is the uncoupled stiffness matrix of the cables. By using eqn (19), eqn (18) takes the form:

$$\tilde{B}\boldsymbol{\delta} = -\mathbf{Q}$$
$$\tilde{B} = \tilde{A}\tilde{K}\tilde{A}^{\mathrm{T}}. \quad (20)$$

This is the case of ordinary reticulated shells which is discussed extensively in the literature.

In the case of underconstrained nets, inequality (2) implies that the rank of matrix $\tilde{B}$ is r and $\tilde{B}$ can be partitioned in accordance with matrix $\tilde{A}$ into:

$$\tilde{B} = \begin{bmatrix} \tilde{B}^0 & \tilde{B}^g \\ \tilde{B}^d & \tilde{B}^e \end{bmatrix} \quad (21)$$

in which $\tilde{B}^0$ is an $r \times r$ with a determinant different from zero and $\tilde{B}^g$, $\tilde{B}^d$ and $\tilde{B}^e$ are the appropriate $r \times (n-r)$, $(n-r) \times r$ and $(n-r) \times (n-r)$ matrices composed of the rest of the elements of matrix $\tilde{B}$ accordingly. Also the elements of these matrices are inter-related:

$$\tilde{B}^d = \tilde{V}\tilde{B}^0$$
$$\tilde{B}^c = \tilde{V}\tilde{B}^q$$
$$\tilde{B}^q = \tilde{B}^0\tilde{V}^{\mathrm{T}}. \tag{22}$$

Because of the nature of matrix $\tilde{B}$ of an underconstrained cable net, it is possible to distinguish between two loading cases. The so-called "fitted load" case and the "non-fitted load". In the case of a "fitted load" the first elements of the load vector $\mathbf{Q}$, denoted by $\mathbf{Q}^t$, are related to the last $(n-r)$ elements of vector $\mathbf{Q}$ denoted by $\mathbf{Q}^b$:

$$\mathbf{Q}^b = \tilde{V}\mathbf{Q}^t. \tag{23}$$

Where the load is a "fitted load" and eqn (23) is satisfied, eqn (20) is a consistent set of equations. This fact implies that only loads satisfying eqn (23) can be sustained by the net in its prestressed configuration and the nodal displacements caused by it are small and due to the elasticity of the net elements. By using matrix analysis and eqns (21) and (19), the change of the internal forces in the "fitted load" loading cases take the form:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}^t \\ \mathbf{F}^b \end{bmatrix}$$
$$\mathbf{F}^t = -\tilde{K}(\tilde{A}^0)^{\mathrm{T}}(\tilde{B}^0)^{-1}\mathbf{Q}^t; \quad \mathbf{F}^b = -\tilde{K}(\tilde{A}^q)^{\mathrm{T}}(\tilde{B}^0)^{-1}\mathbf{Q}^t. \tag{24}$$

Because of the singularity of matrix $\tilde{A}$, the nodal displacements of underconstrained cable nets cannot be determined by using eqn (19). Moreover, there is the possibility of rigid body movement of the members without inducing any forces in them. In the case where the nodal displacements take the form:

$$\boldsymbol{\delta}_l = \begin{bmatrix} -\tilde{V}^{\mathrm{T}}\boldsymbol{\delta}_l^b \\ \boldsymbol{\delta}_l^b \end{bmatrix} \tag{25}$$

in which the order of vector $\boldsymbol{\delta}_l^b$ is $(n-r)$, eqn (19) takes the form:

$$\mathbf{F} = \tilde{K}\tilde{A}^{\mathrm{T}}\boldsymbol{\delta}_l = \tilde{K}(-(\tilde{A}^0)^{\mathrm{T}}\tilde{V}^{\mathrm{T}} + (\tilde{A}^d)^{\mathrm{T}} - (\tilde{A}^q)^{\mathrm{T}}\tilde{V}^{\mathrm{T}} + (\tilde{A}^e)^{\mathrm{T}})\boldsymbol{\delta}_l^b = 0. \tag{26}$$

Equation (26) implies that the nodal displacements given by eqn (25) are due to rigid body movement of the net members and are not associated with a change in the internal forces. In this case the nodal displacements caused by the "fitted load" can be found by considering their effect on the elements of matrix $\tilde{A}$. Because of the nodal displacements the equilibrium matrix changes from $\tilde{A}$ to a new equilibrium matrix denoted by $\tilde{G}$. The elements of matrix $\tilde{G}$ are a function of the node coordinates and the nodal displacements. The elements of matrix $\tilde{G}$ are investigated by studying the change of the configuration of a typical member of the cable net shown in Fig. 2. The inclination of the member changes from:

$$\cos\alpha_x^0 = \frac{x_{ji}}{l_0}; \quad l_0 = \sqrt{x_{ji}^2 + y_{ji}^2 + z_{ji}^2} x_{ji} = x_j - x_i; \quad y_{ji} = y_j - y_i; \quad z_{ji} = z_j - z_i \tag{27}$$

to

$$\cos\alpha_x = \frac{x_{ji} + u_j^x - u_i^x}{\sqrt{(x_{ji} + u_i^x)^2 + (y_{ji} + u_j^y - u_i^y)^2 + (z_{ji} + u_j^z - u_i^z)^2}}. \tag{28}$$

In the case of small displacements, eqn (28) takes the form

$$\cos\alpha_x = \cos\alpha_x^0 + \frac{l_0^2 - x_{ji}}{l_0^3}(u_j^x - u_i^x) + \frac{x_{ji}y_{ji}}{l_0^3}(u_j^y - u_i^y) + \frac{u_{ji}z_{ji}}{l_0^3}(u_j^z - u_i^z). \tag{29}$$

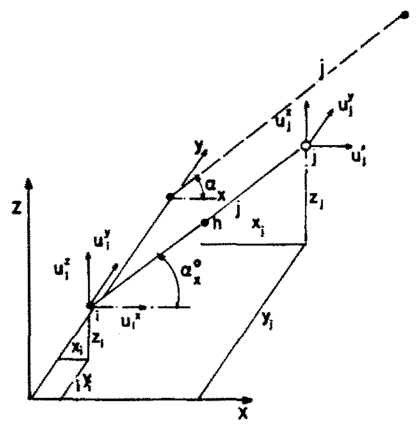

Fig. 2. The distortion of a typical element.

Equation (29) indicates that the change in $\cos \alpha_x^0$ is a linear combination of the nodal displacements.

Equation (29) indicates that matrix $\tilde{G}$ can be written in the following form:

$$\tilde{G} = \tilde{A} + \tilde{O} \tag{30}$$

in which the elements of matrix $\tilde{O}$ are a linear combination of the nodal displacements associated with the last three terms on the right-hand side of eqn (29). By using eqn (30), eqn (17) takes the form:

$$(\tilde{A} + \tilde{O})(\mathbf{P} + \mathbf{F}) + \mathbf{Q} = 0. \tag{31}$$

By using eqn (1), eqn (31) takes the form:

$$\tilde{A}\mathbf{F} + \mathbf{W} + \mathbf{Q} = 0 \tag{32}$$

in which

$$\mathbf{W} = \tilde{O}(\mathbf{P} + \mathbf{F}). \tag{33}$$

The solution of eqn (32) is assumed to take the form:

$$\boldsymbol{\delta} = \boldsymbol{\delta}_s + \boldsymbol{\delta}_l \tag{34}$$

in which the $\boldsymbol{\delta}_s$ are the displacements associated with the change in the internal forces in the members of the net and the $\boldsymbol{\delta}_l$, given by eqn (25), are the displacements associated with rigid body movement of the members of the net which do not cause any change in these internal forces. $\boldsymbol{\delta}_s$ takes the form:

$$\boldsymbol{\delta}_s = \begin{bmatrix} \boldsymbol{\delta}_s^t \\ \mathbf{0} \end{bmatrix}. \tag{35}$$

$\boldsymbol{\delta}_s^t$ is composed of r elements and the last $(n-r)$ elements of vector $\boldsymbol{\delta}_s$ are equal to zero.

By using eqn (19), the change in the internal forces associated with the nodal displacements given by eqn (33) takes the form:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}^t \\ \mathbf{F}^b \end{bmatrix}$$

$$\mathbf{F}^t = \tilde{K}(\tilde{A}^0)^{\mathrm{T}}\boldsymbol{\delta}_s^t; \quad \mathbf{F}^b = \tilde{K}(\tilde{A}^g)^{\mathrm{T}}\boldsymbol{\delta}_s^t. \tag{36}$$

In practical cases of "fitted load" the contribution of the first term of eqn (32) is much larger than the contribution of the second term. Because of the fact that the elements of matrix $\tilde{A}$ are not independent it is possible to divide eqn (32) into two sets of equations, a set of r linear equations of the form:

$$\mathbf{B}^0\boldsymbol{\delta}_s^t = -\mathbf{Q}^t \tag{37}$$

and a set of $(n-r)$ equations of the form:

$$\mathbf{W}^b - \tilde{V}\mathbf{W}^t = 0, \tag{38}$$

in which $\mathbf{W}^t$ and $\mathbf{W}^b$ are the first r and the last $(n-r)$ elements of vector $\mathbf{W}$. By using eqn (37), $\boldsymbol{\delta}_s^t$ takes the form:

$$\boldsymbol{\delta}_s^t = -(\tilde{B}^0)^{-1}\mathbf{Q}^t. \tag{39}$$

By using eqn (24) and by using $\boldsymbol{\delta}_s^t$ and eqn (36), $\mathbf{F}$ can be predicted and $\boldsymbol{\delta}_l^b$ can be found by using eqn (38) by means of linear algebra only.

In the cases where the load does not satisfy eqn (23), the "non-fitted load", eqn (18) is not a consistent set of equations. This fact indicates that "non-fitted load" will cause a large change in the geometrical configuration of the cable net until equilibrium is established at all nodes. Also in this case the nodal displacements can be seen as composed of nodal displacements caused by rigid body movement of the members and caused by the deformation of the members as given by eqn (34). In the case of "non-fitted load" the nodal displacements associated with rigid body movement of the members of the cable net are much larger than the nodal displacements caused by the deformation of the members

$$\boldsymbol{\delta}_l \gg \boldsymbol{\delta}_s. \tag{40}$$

In this case the effect of $\boldsymbol{\delta}_s$ on the elements of matrix $\tilde{O}$ can be ignored:

$$\tilde{O} = \tilde{O}(\delta_l). \tag{41}$$

The large nodal displacements affect the relationship between the nodal displacements and the change in the internal forces and eqn (19) takes the form:

$$\mathbf{F} = \tilde{K}((\tilde{A})^{\mathrm{T}} + 0.5(\tilde{O})^{\mathrm{T}})\boldsymbol{\delta}. \tag{42}$$

By using eqn (42), eqn (31) takes the form:

$$\tilde{B}\boldsymbol{\delta} + \tilde{O}\mathbf{P} + 0.5\tilde{A}\tilde{K}(\tilde{O})^{\mathrm{T}}\boldsymbol{\delta} + \tilde{O}\tilde{K}(\tilde{A})^{\mathrm{T}}\boldsymbol{\delta} + 0.5\tilde{O}\tilde{K}(\tilde{O})^{\mathrm{T}}\boldsymbol{\delta} + \mathbf{Q} = 0. \tag{43}$$

Equation (43) is the cable net equation. The first two terms are a linear combination of the nodal displacements whereas the third and the fourth are of second order and the fifth one is of third order.

By using eqns (25), (26) and (35), eqn (43) can be divided into two sets of equations. The first set is a set of r equations which takes the form:

$$\tilde{B}^0\boldsymbol{\delta}_s^t + \mathbf{W}_1^t + \mathbf{W}_2^t + \mathbf{W}_3^t + \mathbf{W}_4^t + \mathbf{Q}^t = 0 \tag{44}$$

in which

$$\begin{aligned} \mathbf{W}_1 &= \tilde{O}\mathbf{P} \\ \mathbf{W}_2 &= 0.5\tilde{A}\tilde{K}(\tilde{O})^{\mathrm{T}}\boldsymbol{\delta}_l \\ \mathbf{W}_3 &= \tilde{O}\tilde{K}\tilde{A}^{\mathrm{T}}\boldsymbol{\delta}_s \\ \mathbf{W}_4 &= 0.5\tilde{O}\tilde{K}(\tilde{O})^{\mathrm{T}}\boldsymbol{\delta}_l. \end{aligned} \tag{45}$$

Here $\mathbf{W}_1^t$, $\mathbf{W}_2^t$, $\mathbf{W}_3^t$, $\mathbf{W}_4^t$ and $\mathbf{W}_1^b$, $\mathbf{W}_2^b$, $\mathbf{W}_3^b$, $\mathbf{W}_4^b$ are composed of the first r elements and the last $(n-r)$ elements of the appropriate vectors. The second set of equations is a set of $(n-r)$ equations which take the form:

$$\mathbf{W}_1^b - \tilde{V}\mathbf{W}_1^t + \mathbf{W}_3^b - \tilde{V}\mathbf{W}_3^t + \mathbf{W}_4^b - \tilde{V}\mathbf{W}_4^t + \mathbf{Q}^b - \tilde{V}\mathbf{Q}^t = 0. \tag{46}$$

In practical cases the contribution of $\mathbf{W}_3^t$ in eqn (44) can be ignored and $\boldsymbol{\delta}_s^t$ takes the form:

$$\boldsymbol{\delta}_s^t = -(\tilde{B}^0)^{-1}(\mathbf{Q}^t + \mathbf{W}_1^t + \mathbf{W}_2^t + \mathbf{W}_4^t). \tag{47}$$

By using eqn (47), $\boldsymbol{\delta}_l^b$ can be found by eqns (46) which take the form of a set of $(n-r)$ fourth-order algebraic equations. Equations (47) and (46) can be analysed by using an iteration in which $\boldsymbol{\delta}_l$ is first analysed by using eqn (46) assuming $\boldsymbol{\delta}_s^t = 0$ then $\boldsymbol{\delta}_s^t$ is predicted by using eqn (46) considering the new $\boldsymbol{\delta}_s^t$. The method can be carried out until satisfactory convergence is achieved. In this case at every step of the analysis, eqn (46) is a set of $(n-r)$ third-order equations.

In the case of the cable net shown in Fig. 1, where all cables have a cross-section of 1 cm^2, matrix $\tilde{B}$ takes the form:

$$\tilde{B} = \begin{bmatrix} \tilde{B}_1 & \tilde{B}_2 \\ \tilde{B}_2^T & \tilde{B}_1 \end{bmatrix}$$

$$\tilde{B}_1 = \begin{bmatrix} A & 0 & -B & 0 & -C & D & 0 & 0 & 0 \\ 0 & 0 & -F & 0 & 0 & 0 & 0 & 0 & 0 \\ -B & E & H & 0 & I & -J & 0 & 0 & 0 \\ 0 & -F & 0 & E & 0 & 0 & 0 & G & 0 \\ -C & 0 & I & 0 & K & 0 & -C & 0 & I \\ D & 0 & -J & 0 & 0 & L & -D & 0 & -J \\ 0 & 0 & 0 & 0 & -C & -D & A & 0 & B \\ 0 & 0 & 0 & -G & 0 & 0 & 0 & E & -F \\ 0 & 0 & 0 & 0 & -I & -J & B & -F & H \end{bmatrix}$$

$$\tilde{B}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -G & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -F \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -F & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{48}$$

$A = 8.95\,\mathrm{kN\,mm^{-1}}$; $B = 2.131\,\mathrm{kN\,mm^{-1}}$; $C = 4.926\,\mathrm{kN\,mm^{-1}}$;

$D = 0.611\,\mathrm{kN\,mm^{-1}}$; $E = 8.576\,\mathrm{kN\,mm^{-1}}$; $F = 1.788\,\mathrm{kN\,mm^{-1}}$;

$G = 5.00\,\mathrm{kN\,mm^{-1}}$; $H = 1.540\,\mathrm{kN\,mm^{-1}}$; $I = 0.615\,\mathrm{kN\,mm^{-1}}$;

$J = 0.076\,\mathrm{kN\,mm^{-1}}$; $K = 9.922\,\mathrm{kN\,mm^{-1}}$; $L = 1.047\,\mathrm{kN\,mm^{-1}}$.

The non-zero elements of matrix $\tilde{O}$ are given in Table 1. (The nodal displacements are in metres.)

The loading case in which six vertical point loads of 20 kN are applied to all the nodes is found by using eqn (23) as the "fitted load". The vertical displacement at each of the six nodes found by using eqns (31) and (38) is only 0.2 cm at nodes A, C, D and F and 0.24

Table 1. The non-zero elements of matrix $\tilde{O}$ for the cable net shown in Fig. 1.

$0_{11} = +0.224\delta_1$	$0_{14} = +0.029\delta_1 + 0.077\delta_3$	$0_{15} = 0.004(\delta_1 - \delta_5) - 0.03(\delta_6 - \delta_3)$	$0_{18} = 0.25(\delta_{10} - \delta_1)$
$0_{21} = +0.045\delta_2 + 0.09\delta_3$	$0_{24} = +0.234\delta_2$	$0_{25} = 0.248(\delta_2 - \delta_{17})$	
$0_{31} = +0.179\delta_3 + 0.09\delta_2$	$0_{34} = +0.205\delta_3 + 0.077\delta_1$	$0_{35} = 0.244(\delta_2 - \delta_{17}) - 0.03(\delta_5 - \delta_1)$	$0_{38} = 0.25(\delta_3 - \delta_{12})$
$0_{49} = -0.25(\delta_{16} - \delta_7)$	$0_{413} = +0.249(\delta_4 - \delta_{14})$	$0_{414} = +0.234\delta_1$	$0_{417} = +045\delta_4 - 0.09\delta_{18}$
$0_{52} = +0.244\delta_5$	$0_{55} = +0.004(\delta_5 - \delta_1) + 0.03(\delta_1 - \delta_3)$	$0_{56} = 0.004(\delta_5 - \delta_7) - 0.03(\delta_6 - \delta_9)$	$0_{59} = 0.25(\delta_6 - \delta_9)$
$0_{62} = +0.179\delta_6 + 0.09\delta_{17}$	$0_{65} = +0.244(\delta_6 - \delta_3) + 0.03(\delta_5 - \delta_1)$	$0_{66} = 0.244(\delta_6 - \delta_9) - 0.03(\delta_5 - \delta_7)$	$0_{610} = 0.25(\delta_{17} - \delta_{16})$
$0_{73} = +0.224\delta_7$	$0_{76} = +0.004(\delta_7 - \delta_5) + 0.03(\delta_6 - \delta_9)$	$0_{77} = +0.029\delta\delta_7 - 0.077\delta_9$	
$0_{83} = +0.045\delta_8 + 0.098\delta_9$	$0_{86} = +0.248(\delta_8 - \delta_{17})$	$0_{87} = +0.234\delta_8$	$0_{810} = 0.25(\delta_9 - \delta_{18})$
$0_{93} = +0.179\delta_9 + 0.09\delta_8$	$0_{96} = +0.244(\delta_9 - \delta_6) + 0.03(\delta_5 - \delta_7)$	$0_{97} = +0.205\delta_9 - 0.077\delta_7$	
$0_{108} = +0.25(\delta_{10} - \delta_1)$	$0_{1011} = +0.29\delta_{10} + 0.077\delta_{12}$	$0_{1012} = +0.004(\delta_{13} - \delta_{10}) - 0.03(\delta_{15} - \delta_{12})$	$0_{1015} = +0.224\delta_{10}$
$0_{1111} = +0.234\delta_{11}$	$0_{1112} = +0.247(\delta_{11} - \delta_{14})$	$0_{1115} = +0.045\delta_{11} - 0.09\delta_{12}$	
$0_{118} = +0.25(\delta_{12} - \delta_3)$	$0_{1211} = +0.205\delta_{12} + 0.077\delta_{10}$	$0_{1212} = 0.224(\delta_{12} - \delta_{15}) - 0.03(\delta_{11} - \delta_{10})$	$0_{1215} = +0.179\delta_{12} - 0.09\delta_{11}$
$0_{139} = +0.25(\delta_{13} - \delta_{10})$	$0_{1312} = +0.004(\delta_{13} - \delta_{10})$	$0_{1313} = +0.004(\delta_{16} - \delta_{13}) - 0.03(\delta_{18} - \delta_{15})$	$0_{1316} = +0.224\delta_{14}$
$0_{1412} = 0.248(\delta_{14} - \delta_{17})$	$0_{1413} = +0.248(\delta_{14} - \delta_9)$	$0_{1416} = +0.045\delta_{14} - 0.09\delta_{15}$	
$0_{159} = +0.25(\delta_{18} - \delta_9)$	$0_{1512} = +0.244(\delta_{15} - \delta_{16}) + 0.03(\delta_{13} - \delta_{10})$	$0_{1513} = +0.244(\delta_{18} - \delta_{15}) - 0.03(\delta_{13} - \delta_{16})$	$0_{1516} = +0.179\delta_{15} - 0.09\delta_{14}$
$0_{1610} = +0.25(\delta_{16} - \delta_7)$	$0_{1613} = +0.004(\delta_{16} - \delta_{13}) + 0.03(\delta_{15} - \delta_{19})$	$0_{1614} = +0.029\delta_{16} - 0.077\delta_{18}$	$0_{1617} = +0.224\delta_4$
$0_{172} = +0.045\delta_{19} + 0.09\delta_6$	$0_{175} = +0.249(\delta_{17} - \delta_2)$	$0_{176} = -0.248(\delta_8 - \delta_{17})$	
$0_{1810} = +0.25(\delta_{18} - \delta_9)$	$0_{1813} = +0.244(\delta_{18} - \delta_{15}) + 0.03(\delta_{13} - \delta_{16})$	$0_{1814} = +0.205\delta_{18} - 0.077\delta_{16}$	$0_{1817} = +0.179\delta_{19} - 0.09\delta_4$

at nodes B and E. In the case where the vertical point load of 20.0 kN is applied to nodes A and F only, the vertical displacement at these joints found by using eqns (46) and (47) is 6 cm. The large nodal displacements associated with the "non-fitted load" compared with the "fitted load" are typical of underconstrained cable nets.

THE FREE DYNAMIC RESPONSE

In the case of free dynamical response of the net, eqn (17) takes the form:

$$\tilde{A}(\mathbf{P}+\mathbf{F})+\tilde{M}\ddot{\delta} = 0 \tag{49}$$

where $\tilde{M}$ is a diagonal matrix of the net nodal masses and $\ddot{\delta}$ is the vector of the nodal acceleration. By using eqn (1), eqn (49) takes the form:

$$\tilde{B}\delta+\tilde{M}\ddot{\delta} = 0. \tag{50}$$

In the case of a fully constrained cable net, eqn (50) is a consistent set of equations. It is the case of undamped systems with multiple degrees of freedom which is discussed extensively in the literature. The free vibration frequencies and the modes of displacements are found by considering the eigenvalues and eigenfunction of eqn (50).

In the case of underconstrained cable nets, eqn (49) is not a consistent set of equations. This fact indicates that the change in the net configuration due to dynamical displacements should be considered. By using eqns (43), eqn (49) takes the form:

$$\tilde{B}\delta+\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_4+\tilde{M}\ddot{\delta} = 0. \tag{51}$$

In the case of very small dynamical displacements where the second and third order of the displacements can be ignored, the solution of eqn (51) takes the form:

$$\delta = \sum_i^n \begin{bmatrix} -\tilde{V}^{\mathrm{T}}\delta^b_{M_i}+\delta_{N_i} \\ \delta^b_{M_i} \end{bmatrix} \cos\,(w_i t+\psi_i) \tag{52}$$

in which $\delta^b_{M_i}$ and δ_{N_i} are vectors of order $(n-r)$ and r of the maximum amplitude of the nodal displacements, and w_i and ψ_i are the appropriate angular frequency and phase angle of the mode. The frequencies of response of the net can be divided into two groups, high frequencies of vibration w^h_i and low frequencies of vibration w^l_i where $(w^h_i)^2 \gg (w^l_i)^2$. Where high frequencies of vibrations are considered, eqn (51) can be practically divided into two sets of equations, a set of r equations of the form:

$$\tilde{B}^0\delta_{N_i}-(w^h_i)^2(\tilde{M}^0\delta_{N_i}-\tilde{M}^0\tilde{V}^{\mathrm{T}}\delta^b_{M_i}) = 0 \tag{53}$$

in which $\tilde{M}^0$ is the matrix composed of the elements in the first r rows and columns of matrix $\tilde{M}$; and a set of $(n-r)$ equations of the form:

$$(\tilde{M}^e+\tilde{V}\tilde{M}^0\tilde{V}^{\mathrm{T}})\delta^b_{M_i}-\tilde{V}\tilde{M}^0\delta_{N_i} = 0 \tag{54}$$

in which $\tilde{M}^e$ is composed of the last $(n-r)\times(n-r)$ columns and rows of matrix $\tilde{M}$. Equation (54) implies:

$$\delta^b_{M_i} = \tilde{D}^0\delta_{N_i}; \quad \tilde{D}^0 = (\tilde{M}^e+\tilde{V}\tilde{M}^0\tilde{V}^{\mathrm{T}})^{-1}\tilde{V}\tilde{M}^0. \tag{55}$$

By using eqn (55), eqn (53) takes the form:

$$\tilde{B}^0\delta_{N_i}-(w^h_i)^2(\tilde{M}^0-\tilde{M}^0\tilde{V}^{\mathrm{T}}\tilde{D}^0)\delta_{N_i} = 0. \tag{56}$$

The r modes and frequencies of free vibration associated with the high frequencies of response can be found by using eqn (56).

In the case where low frequencies of vibration are considered, eqn (51) can be practically divided into two sets of equations, a set of r equations which takes the form:

$$\tilde{B}^0 \delta_{N_i} = 0 \tag{57}$$

which implies:

$$\delta_{N_i} = 0 \tag{58}$$

and a set of $(n-r)$ equations which by considering eqn (58) takes the form:

$$\mathbf{W}_1^b - \tilde{V}\mathbf{W}_1^t - (w_i^l)^2(\tilde{M}^e + \tilde{V}\tilde{M}^0\tilde{V}^{\mathrm{T}})\delta_{M_i}^b = 0. \tag{59}$$

The $(n-r)$ frequencies and modes of free vibration associated with the low frequencies of response can be found by using eqn (59).

In the case where the nodal displacements are moderately large and the second- and third-order effects can be ignored only where the high frequencies of response are considered, the mode of response in the high frequencies takes the form given by eqn (54) and eqn (59) takes the form:

$$\mathbf{W}_1^b - \tilde{V}\mathbf{W}_1^t + \mathbf{W}_3^b - \tilde{V}\mathbf{W}_3^t + \mathbf{W}_4^b - \tilde{V}\mathbf{W}_4^t + (\tilde{\mathbf{M}}^e + \tilde{V}\tilde{M}^0\tilde{V}^{\mathrm{T}})\ddot{\delta}_l^b = 0. \tag{60}$$

Equation (60) indicates a non-linear response in which the low frequencies of the net response depend on the amplitude of the nodal displacements. It can be predicted by using any of the well-established methods for non-linear dynamics.

In the case where the nodal displacements are large, the second- and third-order effect cannot be ignored where the response associated with low and high frequencies are considered. In this case all the frequencies of response depend on the amplitude of the nodal displacements. In practical cases of large nodal displacements where $\tilde{B}\delta \gg \mathbf{W}_3$ and where eqn (40) is valid, eqn (51) can be divided into two sets of equations, a set of r equations of the form:

$$\tilde{B}^0\delta_s^t + \tilde{M}^0\ddot{\delta}_s^t = -\mathbf{W}_1^t + \mathbf{W}_2^t + \mathbf{W}_4^t - \tilde{M}^0\tilde{V}^{\mathrm{T}}\ddot{\delta}_l^b \tag{61}$$

and a set of $(n-r)$ equations of the form:

$$\mathbf{W}_1^b - \tilde{V}\mathbf{W}_1^t + \mathbf{W}_3^b - \tilde{V}\mathbf{W}_3^t + \mathbf{W}_4^b - \tilde{V}\mathbf{W}_4^t + (\tilde{M}^e + \tilde{V}\tilde{M}^0\tilde{V}^{\mathrm{T}})\ddot{\delta}_l^b - \tilde{V}\tilde{M}^0\ddot{\delta}_s^t = 0. \tag{62}$$

By using eqn (61) and Duhamel's integral, in which the normal modes and frequencies of vibration are the eigenvalues and eigenfunctions of the self-adjoint equation on the left-hand side of eqn (61), the relationship between δ_s^t, δ_l^b and $\ddot{\delta}_l^b$ can be established. The magnitude of δ_l^b can be determined by solving the non-linear free vibration problem given by eqn (62).

In the case of the cable net shown in Fig. 1 there are two modes of low frequencies. In the case where the mass of the net is 32 N s^2 cm^{-1} concentrated at each node, the low vibration frequencies found by using eqn (59) are 2.024 and 2.446 s^{-1}. There are 16 modes of high frequency and the lowest of the high frequencies given by eqn (56) is 7.468 s^{-1}. The large difference in the magnitude of the square of the low and high frequencies of free dynamical response, which in this case is 4.098 and 5.983 compared with 55.779, is typical of underconstrained cable nets.

FORCED VIBRATION

In the case where a dynamical load $\mathbf{Q}$ imposes the nodes of a fully constrained net, eqn (50) takes the form:

$$\tilde{B}\delta+\tilde{M}\ddot{\delta}+\mathbf{Q}=0. \tag{63}$$

This is the case of an undamped system with multiple degrees of freedom which is discussed extensively in the literature.

In the case where the dynamical load $\mathbf{Q}$ imposes the nodes of an underconstrained cable net, eqn (51) takes the form:

$$\tilde{B}\delta+\tilde{O}\mathbf{P}+0.5\tilde{A}\tilde{K}(\tilde{O})^{\mathrm{T}}\delta+\tilde{O}\tilde{K}(\tilde{A})^{\mathrm{T}}\delta+0.5\tilde{O}\tilde{K}(\tilde{O})^{\mathrm{T}}\delta+\tilde{M}\ddot{\delta}+\mathbf{Q}=0. \tag{64}$$

In the case of small nodal displacements where the effect of high order of nodal displacements can be ignored, eqn (64) takes the form:

$$\tilde{B}\delta+\tilde{O}\mathbf{P}+\tilde{M}\ddot{\delta}+\mathbf{Q}=0. \tag{65}$$

In this case the nodal displacements can be found by using Duhamel's integral considering the modes and frequencies of free vibration found earlier. Large nodal displacements are particularly expected at low frequencies of vibration. Only in the cases where the response in these frequencies is eliminated, is the assumption of small nodal displacements valid.

By using Duhamel's integral the elimination of the response in low frequencies is conditioned, in the case where the net is initially at rest, by:

$$\begin{bmatrix} -\tilde{V}^{\mathrm{T}}\delta_l^b \\ \delta_l^b \end{bmatrix}^{\mathrm{T}}\mathbf{Q}=(\delta_l^b)^{\mathrm{T}}(-\tilde{V}\mathbf{Q}^t+\mathbf{Q}^b)=0. \tag{66}$$

Equations (66) and (23) indicate that also in the case of dynamical response, small nodal displacements are conditioned by "fitted load". It should be noted that in the case of forced vibration, this condition is not enough due to the possibility of a resonance between the net responses in high frequencies and of the "fitted load".

In the cases where the load is "non-fitted load" the higher order effect of nodal displacements cannot be ignored. In these cases the nodal displacements take the form given by eqn (34) and the condition given by eqn (40) is valid. In practical cases where $\tilde{B}^0\delta_s^t \gg \mathbf{W}_3^t$, eqn (64) can be divided into two sets of equations, a set of r equations of the form:

$$\tilde{B}^0\delta_s^t+\tilde{M}^0\ddot{\delta}_s^t=-(\mathbf{Q}^t+\mathbf{W}_1^t+\mathbf{W}_2^t+\mathbf{W}_4^t-\tilde{M}^0\tilde{V}^{\mathrm{T}}\ddot{\delta}_l^b) \tag{67}$$

and a set of $(n-r)$ equations of the form:

$$\mathbf{W}_1^b-\tilde{V}\mathbf{W}_1^t+\mathbf{W}_3^b-\tilde{V}\mathbf{W}_3^t+\mathbf{W}_4^b-\tilde{V}\mathbf{W}_4^t+(\tilde{M}^e+\tilde{V}\tilde{M}^0\tilde{V}^{\mathrm{T}})\ddot{\delta}_l^b-(\mathbf{Q}^b-\tilde{V}\mathbf{Q}^t-\tilde{V}\tilde{M}^0\ddot{\delta}_s^t)=0. \tag{68}$$

By considering the eigenvalues and eigenfunctions of the self-adjoint equation on the right-hand side of eqn (67) and by using Duhamel's integral. δ_s^t can be determined as a function of δ_l^b and $\ddot{\delta}_l^b$. In this case, eqn (68) takes the form of $(n-r)$ fourth-order dynamical equations of δ_l^b which can be solved by using the well-established methods for the analysis of non-linear dynamical systems.

CONCLUSIONS

In the previous pages the behaviour of fully constrained and underconstrained cable nets was discussed. It was shown that initial stability is satisfied where the net is prestressable and prestressing induces tension in all cables. It was shown that the response of a fully constrained cable net is similar to an ordinary reticulated shell. In the case of underconstrained cable nets only in the so-called "fitted load" loading cases are the nodal displacements small and due to the elastic rigidity of the members of the cable net. Other loads will cause large nodal displacements due to rigid body movement of the members. A method was proposed to analyse these nodal displacements. The method was extended to consider the dynamical behaviour of the net. It was shown that the free dynamical response of the net is associated with high and low frequencies of vibration. The net response to a dynamical imposed load was investigated. Also in this case only where the dynamical load is a "fitted load" and no resonance with the high free vibration frequencies take place, are the nodal displacements small. In other loading cases the nodal displacements are large and a method was proposed for their analysis.

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